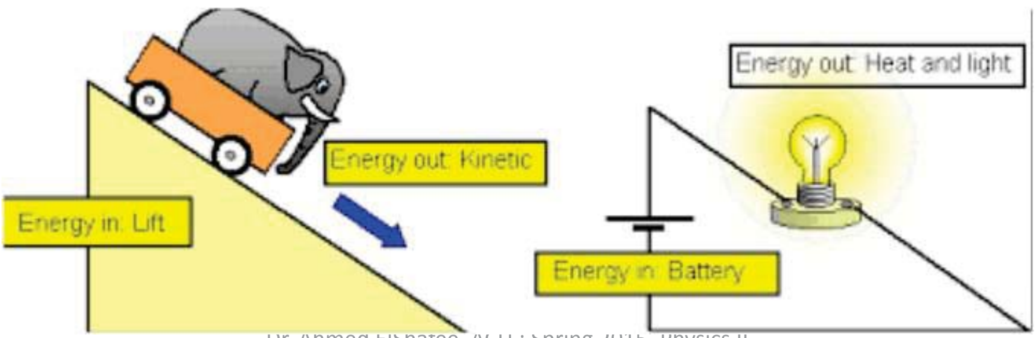
Lecture (05) Electric Potential

By:

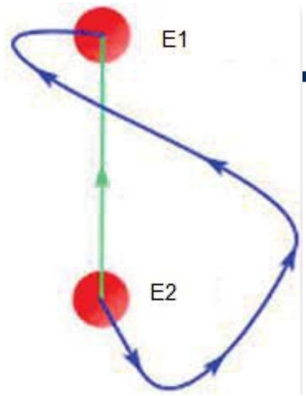
Dr. Ahmed ElShafee

introduction

- We are used to voltage in our lives—a 12-volt car battery, 110 V or 220 V at home, 1.5 volt flashlight batteries, and so on.
- Voltage is the same as electric potential difference between two points.
- Electric potential is defined as the potential energy per unit charge.



Electrostatic Potential Energy and Potential Difference



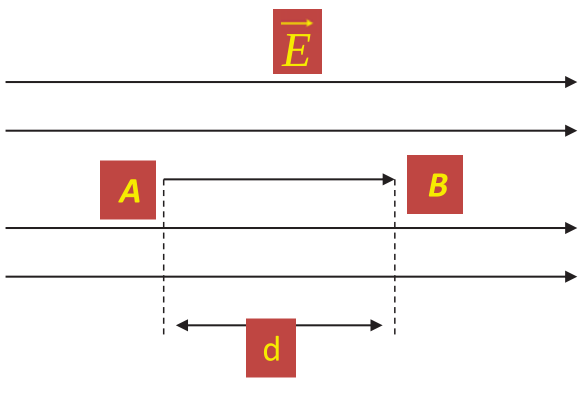
- The Coulomb (Electrostatic) force is a conservative force (A *conservative force* is a *force* with the property that the work done in moving a particle between two points is independent of the taken path.)
- A potential energy function can be defined for any conservative force, including Coulomb force

- As in mechanics, work is

$$w = f \times d$$

- Work done on the positive charge by moving it from A to B

$$w = E \times q \times d$$



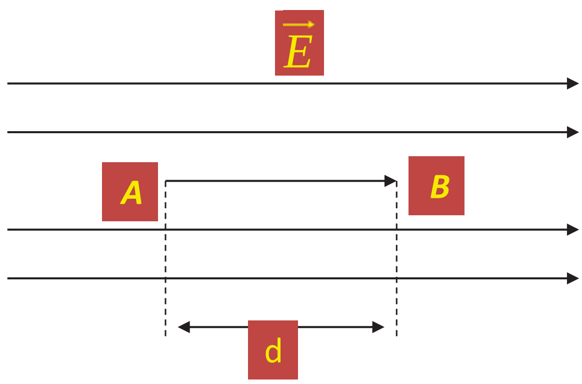
- The work done by a conservative force equals the negative of the change in potential energy, ΔPE

$$\Delta PE(\Delta U) = -W = -fd = -qEd$$

- This equation
  - is valid only for the case of a uniform electric field
  - allows to introduce the concept of electric potential

- Electric potential
- The potential difference between points A and B, V_B-V_A, is defined as the change in potential energy (final minus initial value) of a charge, q, moved from A to B, divided by the charge
- Type equation here.

$$\Delta V = V_B - V_A = \frac{\Delta\, PE\ (\Delta U)}{q} = \frac{U_b - U_a}{q} = -Ed$$

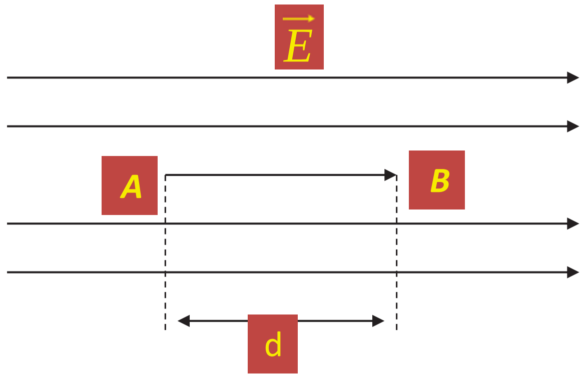


- Electric potential is a scalar quantity
- Electric potential difference is a measure of electric energy per unit charge
- Potential is often referred to as "voltage"

- Electric potential difference is the work done to move a charge from a point A to a point B divided by the magnitude of the charge.
- Thus the SI units of electric potential

$$1v = 1\frac{j}{C}$$

- In other words, 1 J of work is required to move a 1 C of charge between two points that are at potential difference of 1 V



- Units of electric field (N/C) can be expressed in terms of the units of potential (as volts per meter)

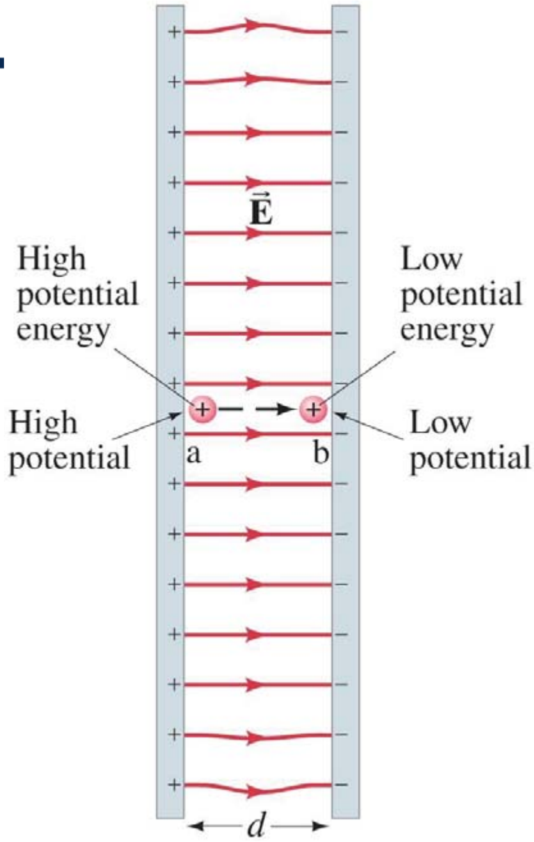
$$\Delta V = V_B - V_A = \frac{\Delta\, PE\ (\Delta U)}{q} = -Ed = -\frac{F}{q}\, d$$

$$\frac{\Delta V}{\mathrm{d}} = -\frac{\mathrm{F}}{\mathrm{q}}$$

$$1\frac{n}{c} = 1\frac{v}{m}$$

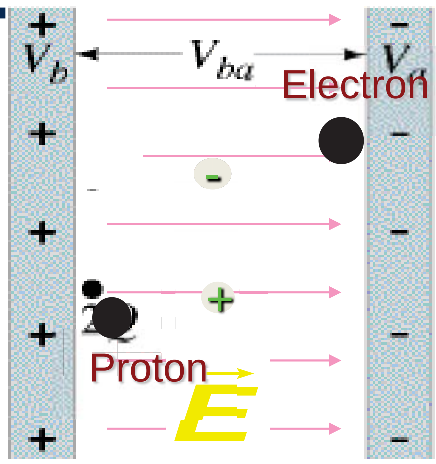
- Because the positive tends to move in the direction of the electric field, work must be done on the charge to move it in the direction, opposite the field

- Thus,
  - A positive charge gains electric potential energy when it is moved in a direction opposite the electric field
  - A negative charge looses electrical potential energy when it moves in the direction opposite the electric field



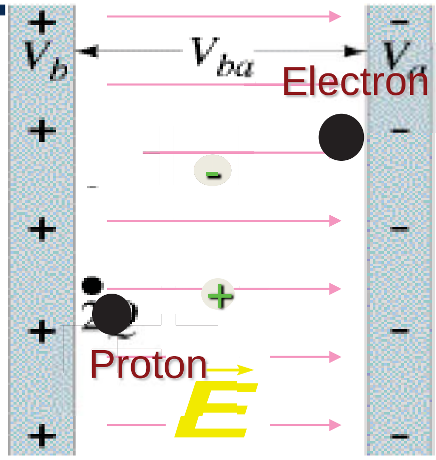
MCQ

- A proton and an electron are in a constant electric field created by oppositely charged plates. You release the proton from the positive side and the electron from the negative side. Which has the larger acceleration?



1. proton
2. electron
3. both feel the same acceleration
4. neither – there is no acceleration
5. they feel the same magnitude acceleration but opposite direction

MCQ



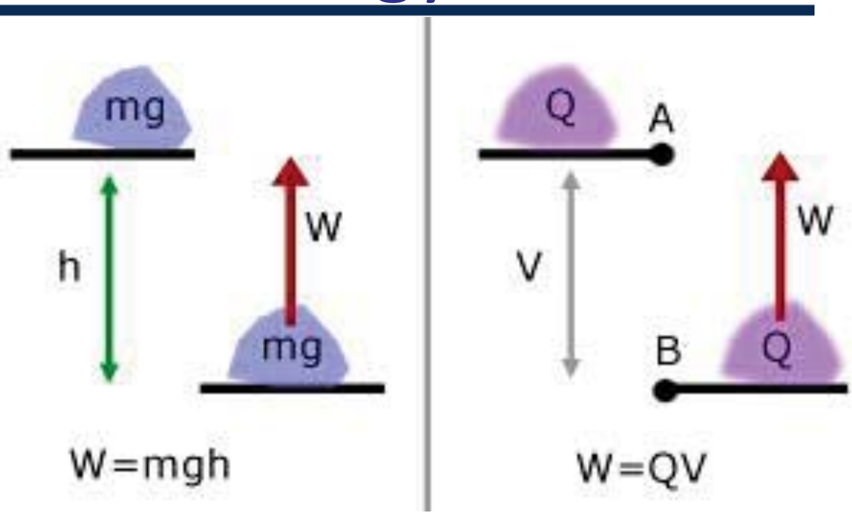
- A proton and an electron are in a constant electric field created by oppositely charged plates. You release the proton from the positive side and the electron from the negative side. Which has the larger acceleration?

1. proton
2. electron
3. both feel the same acceleration
4. neither – there is no acceleration
5. they feel the same magnitude acceleration but opposite direction

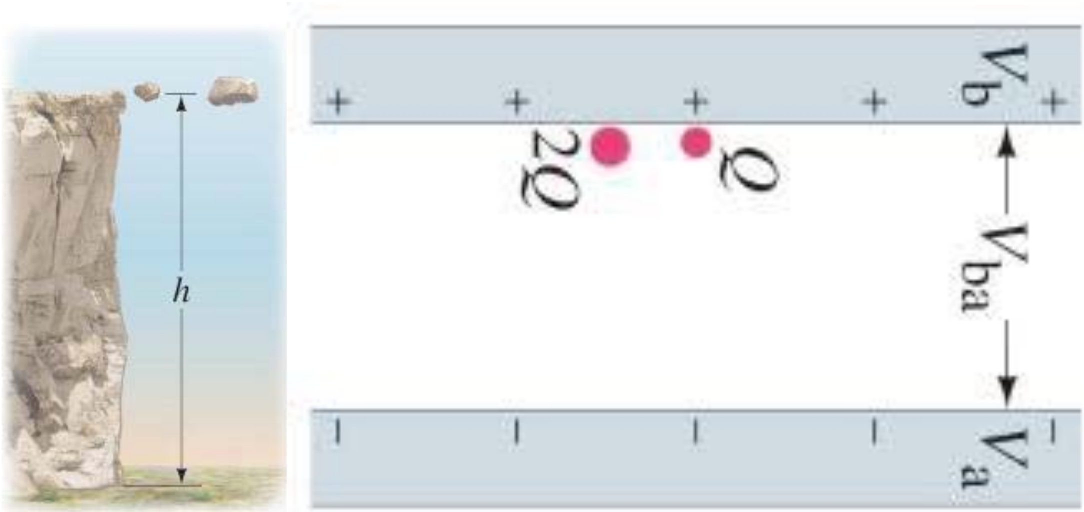
Since $F = ma$ and the electron is much less massive than the proton, the electron experiences the larger acceleration.

Analogy between gravitational and electrical potential energy

- W = f d = E Q d
- Δ V = -E d
- W = -Q Δv

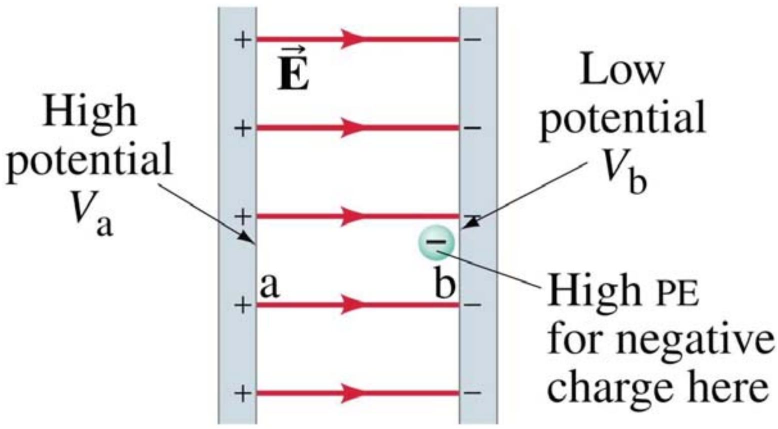


- (a) Two rocks are at the same height.
- The larger rock has more potential energy.
- (b) Two charges have the same electric potential.
- The 2*Q* charge has more potential energy.

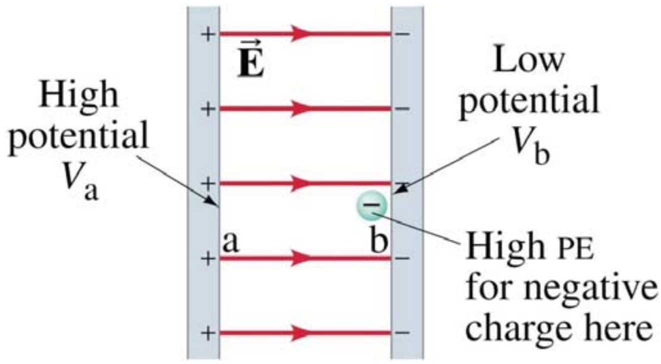


Example 01

- Suppose a negative charge, such as an electron, is placed near the negative plate at point b, as shown here. If the electron is free to move, will its electric potential energy increase or decrease? How will the electric potential change?



- The electron will move towards the positive plate if released, thereby increasing its kinetic energy.
- Its potential energy must therefore decrease. However, it is moving to a region of higher potential V; the potential is determined only by the existing charge distribution and not by the point charge.
- U and V have different signs due to the negative charge.



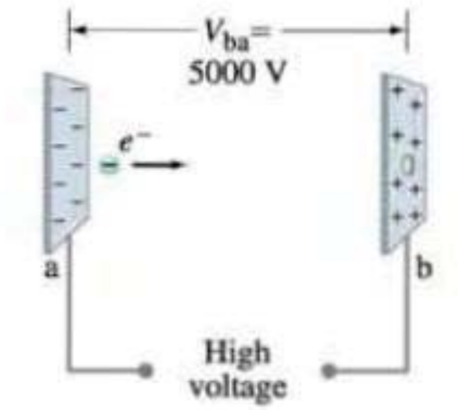
- Electrical sources such as batteries and generators supply a constant potential difference. Here are some typical potential differences, both natural and manufactured:

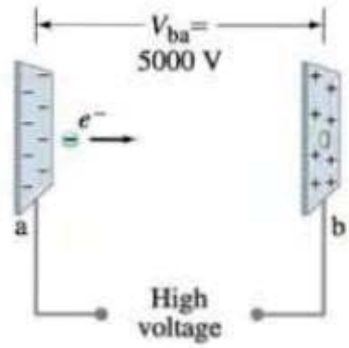
TABLE 23–1 Some Typical Potential Differences (Voltages)

Source	Voltage (approx.)
Thundercloud to ground	10^8 V
High-voltage power line	10^5–10^6 V
Power supply for TV tube	10^4 V
Automobile ignition	10^4 V
Household outlet	10^2 V
Automobile battery	12 V
Flashlight battery	1.5 V
Resting potential across nerve membrane	10^{-1} V
Potential changes on skin (EKG and EEG)	10^{-4} V

Example 02

- Suppose an electron in a cathode ray tube is accelerated from rest through a potential difference $V_b - V_a = V_{ba} = +5000$ V
- (a) What is the change in electric potential energy of the electron?
- (b) What is the speed of the electron ($m = 9.1 \times 10^{-31}$ kg) as a result of this acceleration?





1

- $\Delta U = F\, d = E\, q\, d$
- But $\Delta V = E\, d$
- $\therefore \Delta U = \Delta V \times q = 500 \times -1.6 \times 10^{-19} = -8.0 \times 10^{-16}\, J$

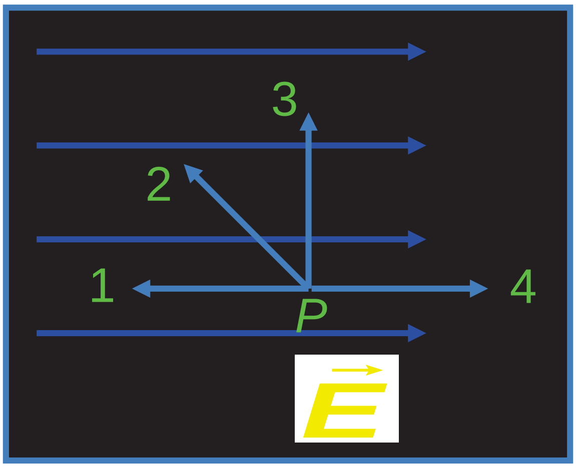
2

- The potential energy lost by the electron becomes kinetic energy *K*. From conservation of energy $\Delta K + \Delta U = 0$, so
- $\Delta K = -\Delta U$
- $\frac{1}{2} m\, v^2 - 0 = -q(v_b - v_a) = -q\, v_{ba}$
- $v = \sqrt{\frac{-2\, q\, v_{ba}}{m}} = \sqrt{\frac{-2 \times -1.6 \times 10^{-19} \times 5000}{9.1 \times 10^{-31}}} = 4.2 \times 10^{7} m/s$

MCQ

- **Which requires the *most work*, to move a positive charge from *P* to points 1, 2, 3 or 4 ? All points are the same distance from *P*.**

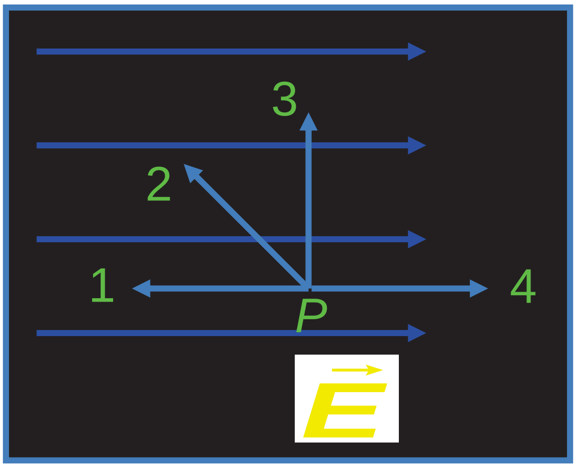
1) $P \rightarrow 1$
2) $P \rightarrow 2$
3) $P \rightarrow 3$
4) $P \rightarrow 4$
5) all require the same amount of work



MCQ

- **Which requires the *most work*, to move a positive charge from *P* to points 1, 2, 3 or 4 ? All points are the same distance from *P*.**

1) $P \rightarrow 1$
2) $P \rightarrow 2$
3) $P \rightarrow 3$
4) $P \rightarrow 4$
5) all require the same amount of work

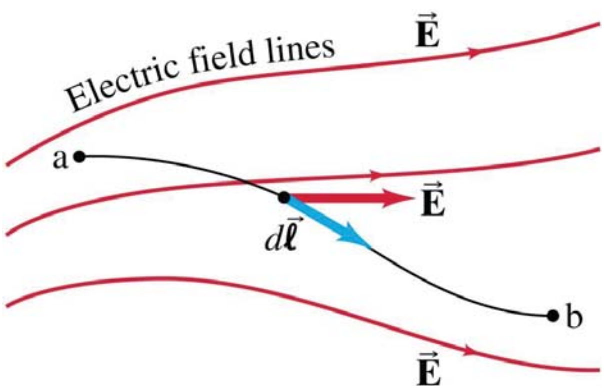


For path #1, you have to push the positive charge *against* the *E* field, which is hard to do. By contrast, path #4 is the easiest, since the field does all the work.

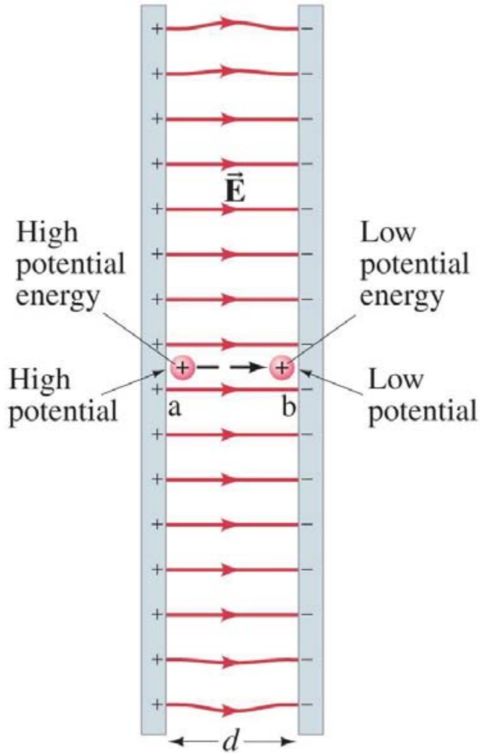
- $\Delta PE\ (\Delta U) = -w = -f\ d$
- $but\ f = k\ \frac{q_1 \times q_2}{d^2}$
- $\Delta PE\ (\Delta U) = -k\frac{q_1 \times q_2}{d} = \frac{-q_1 \times q_2}{4\,\pi\,\varepsilon_0\,d}$

Relation between Electric Potential and Electric Field

- To find V_{ba} in a nonuniform electric field E, we integrate E·dl from point a to point b.
- The general relationship between a conservative force and potential energy
- $U_b\ -\ U_a = -\ \int_a^b f\ dl$
- Substituting the potential difference and the electric field:
- $v_{ba} =\ v_b\ -\ v_a =\ -\ \int_a^b E\ dl$

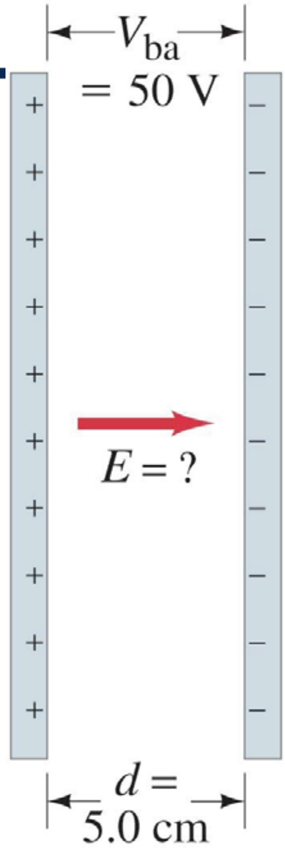


- A simple special case is a uniform field. In Fig., for example, a path parallel to the electric field lines from point a at the positive plate to point bat the negative plate gives (since E and *dl* are in the same direction at each point),
- $v_{ba} = -E\int_a^b dl = -E\ d$

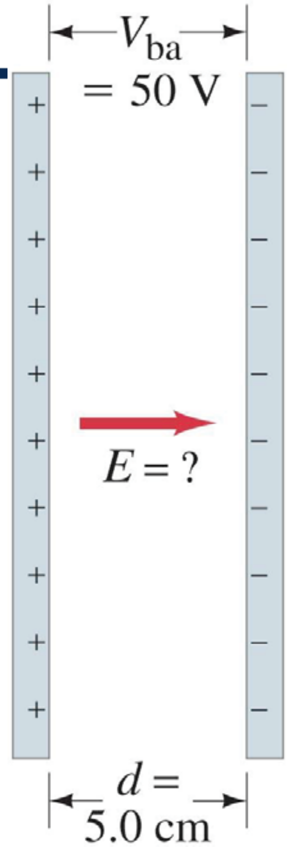


Example 03

- Two parallel plates are charged to produce a potential difference of 50 V. If the separation between the plates is 0.050 m, calculate the magnitude of the electric field in the space between the plates.

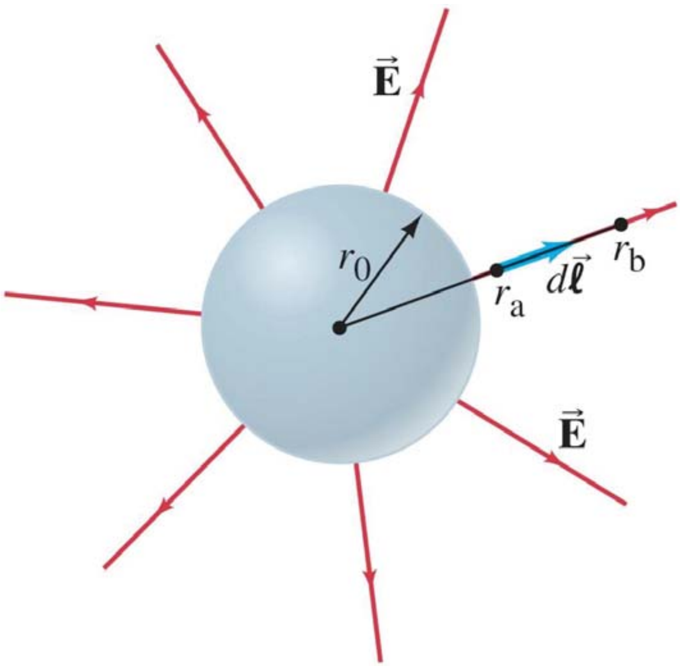


- V = E d
- E = V/d = 50/0.05 = 1000 V/m.



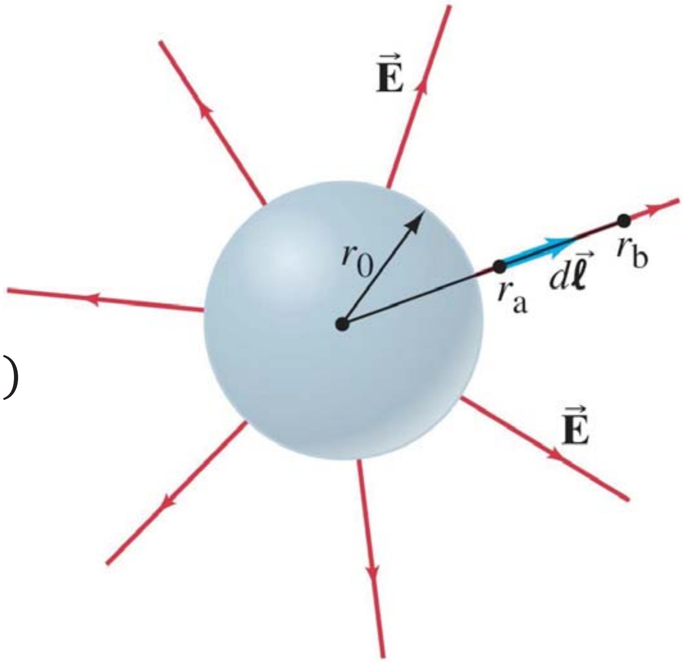
Example 04

- Determine the potential at a distance r from the center of a uniformly charged conducting sphere of radius r_0 for
- (a) $r > r_0$,
- (b) $r = r_0$,
- (c) $r < r_0$.
- The total charge on the sphere is Q.



$a)$

- $for\ r > r_0$
- $E = \frac{1}{4\pi\ \varepsilon_0}\frac{Q}{r^2}$
- $v_b - v_a = -\int_{r_a}^{r_b} E\ dl$
- $= \frac{-Q}{4\ \pi\varepsilon_0}\int_{r_a}^{r_b}\frac{dr}{r^2} = \frac{-Q}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$
- $set\ r_a = r\ and\ r_b = \infty$
- $v_{ba} = \frac{Q}{4\pi\varepsilon_0 r}$
- $v = \frac{Q}{4\pi\varepsilon_0 r}$

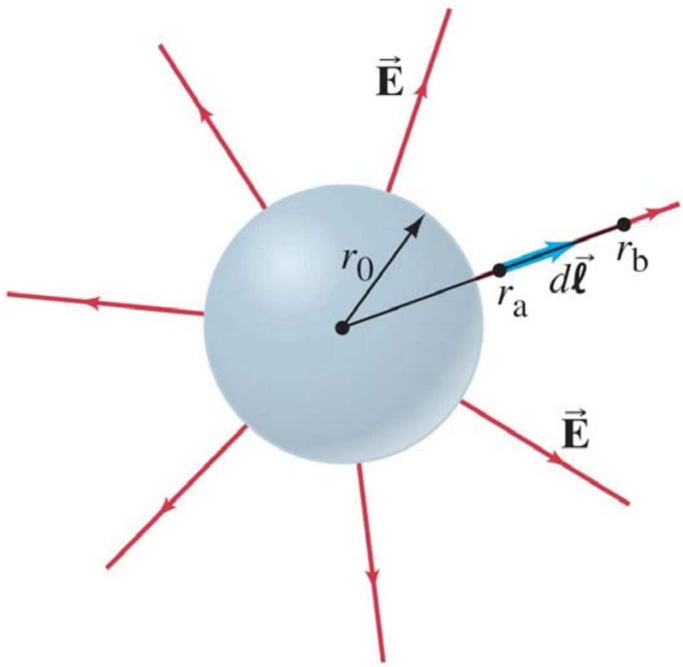


b) $r = r_0$ → conductor has same potential over its surface

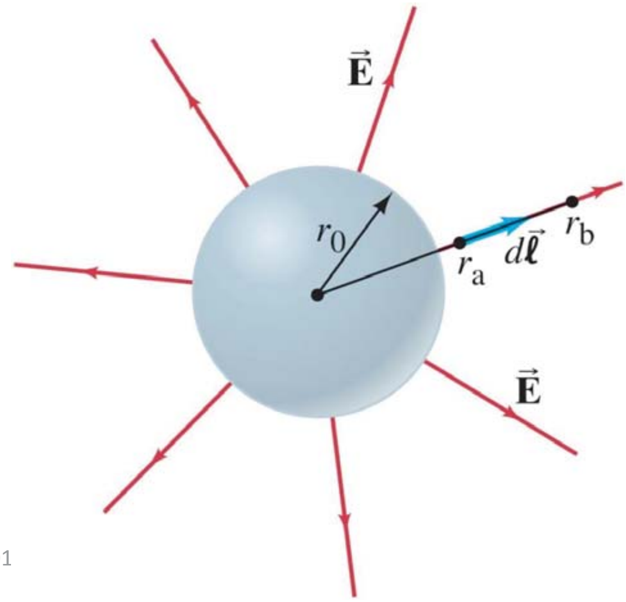
$$v = \frac{Q}{4\pi\varepsilon_0 r_0}$$

C) $r < r_0$ → $conductor\ has\ same\ potential$

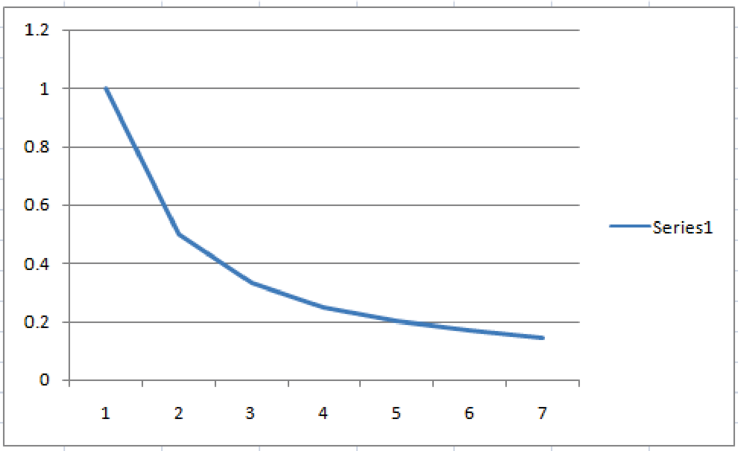
$$v = \frac{Q}{4\pi\varepsilon_0 r_0}$$



- The previous example gives the electric potential as a function of distance from the surface of a charged conducting sphere, which is plotted here, and compared with the electric field:
- $E = \frac{1}{4\pi\, \varepsilon_0} \frac{Q}{r^2}$
- $v = \frac{Q}{4\pi \varepsilon_0 r}$

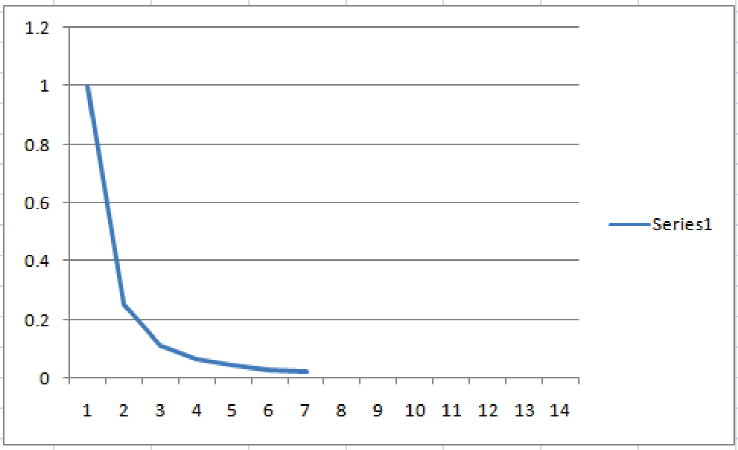


1	1
2	0.5
3	0.333333
4	0.25
5	0.2
6	0.166667
7	0.142857

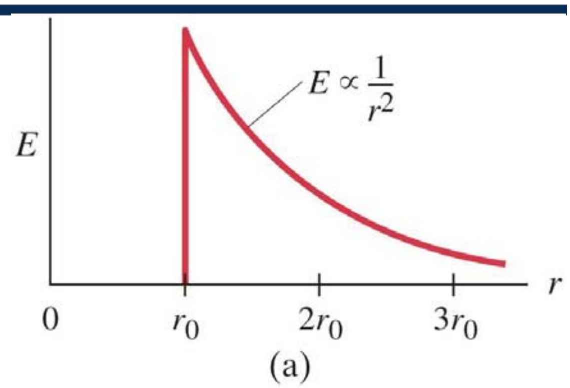


-

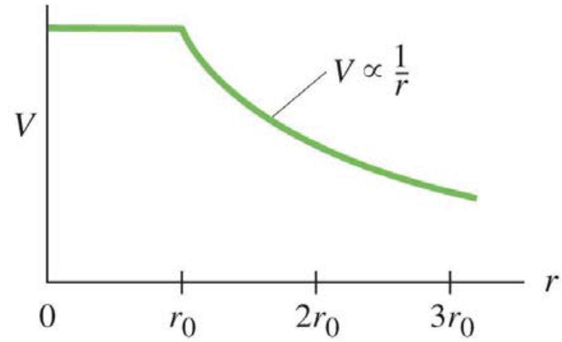
1	1
2	0.25
3	0.111111
4	0.0625
5	0.04
6	0.027778
7	0.020408



- $E = \frac{1}{4\pi\,\varepsilon_0}\frac{Q}{r^2}$

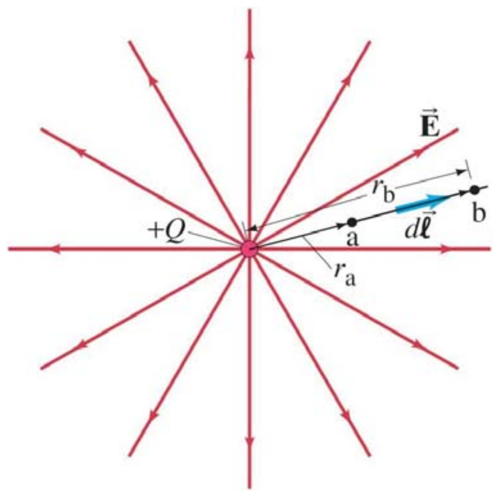


- $v = \frac{Q}{4\pi\varepsilon_0 r_0}$

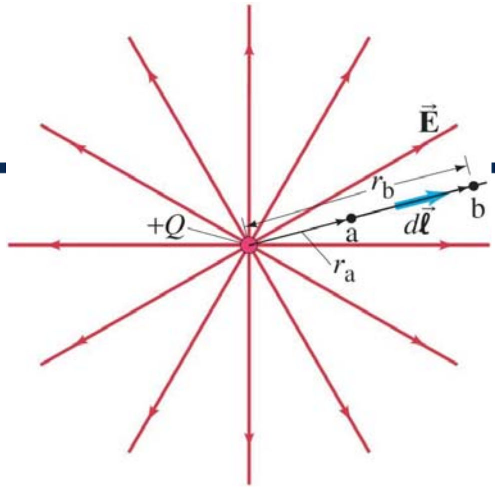


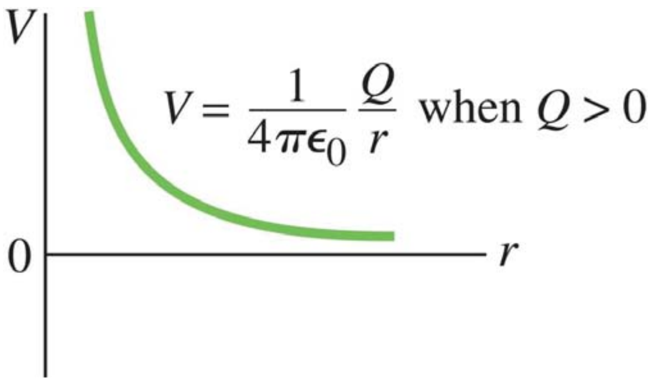
Electric Potential Due to Point Charges

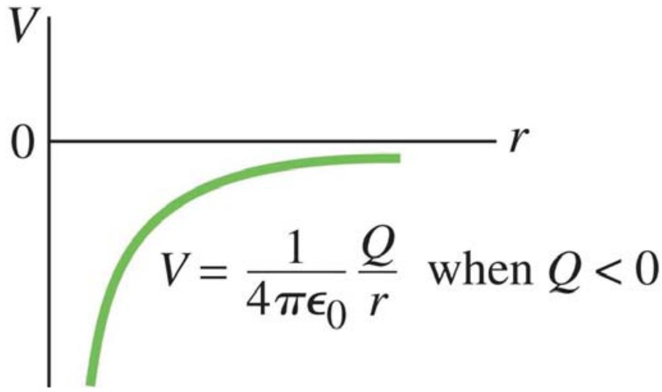
- To find the electric potential due to a point charge, we integrate the field along a field line
- $E = \frac{1}{4\pi\,\varepsilon_0}\frac{Q}{r^2}$
- $v_b - v_a = -\int_{r_a}^{r_b} E\,dl$
- $= \frac{-Q}{4\,\pi\varepsilon_0}\int_{r_a}^{r_b}\frac{dr}{r^2} = \frac{-Q}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$



- Setting the potential to zero at $r = \infty$ gives the general form of the potential due to a point charge:
- $v = \frac{Q}{4\pi\varepsilon_0 r}$







Electrostatic Potential Energy; between two charges

- The potential energy of a charge in an electric potential is

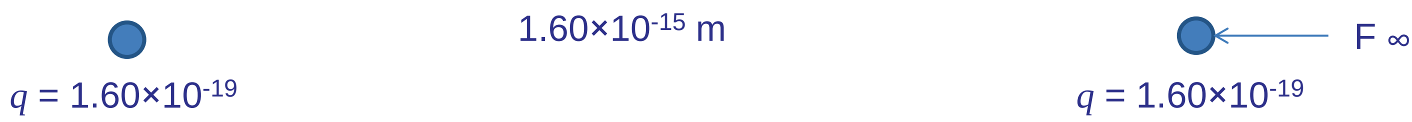
$$U = qV.$$

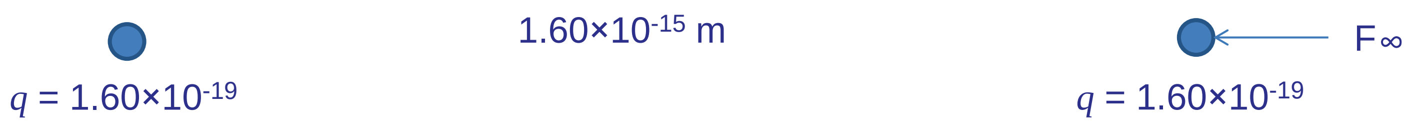
- To find the electric potential energy of two charges, imagine bringing each in from infinitely far away.
- The first one takes no work, as there is no field.
- To bring in the second one, we must do work due to the field of the first one; this means the potential energy of the pair is:

$$U = q_2 V = k\,\frac{q_1 q_2}{r_{12}}$$

Example 05

- Example: Work required to bring two positive charges close together.
- What minimum work must be done by an external force to bring a proton $q = 1.60\times10^{-19}$ C from a great distance away (take $r = \infty$) to a point 1.60×10^{-15} m from another proton?





- The work is equal to the change in potential energy
- $\Delta w = -q\ \Delta v = -q(v_b\ - va) = -q\ \left(\frac{k\ q}{r_b} - \frac{k\ q}{r_a}\right)$
- $r_a = d\ \&\ r_b =\ \infty$
- $\Delta w\ = k\ \frac{q2}{d}$
- $\Delta w = 8.99\ \times 10^9\ \times \frac{(1.6\times10^{-19})^2}{1.6\times10^{-15}} = 2.3\times\ 10^{-13}\ j$

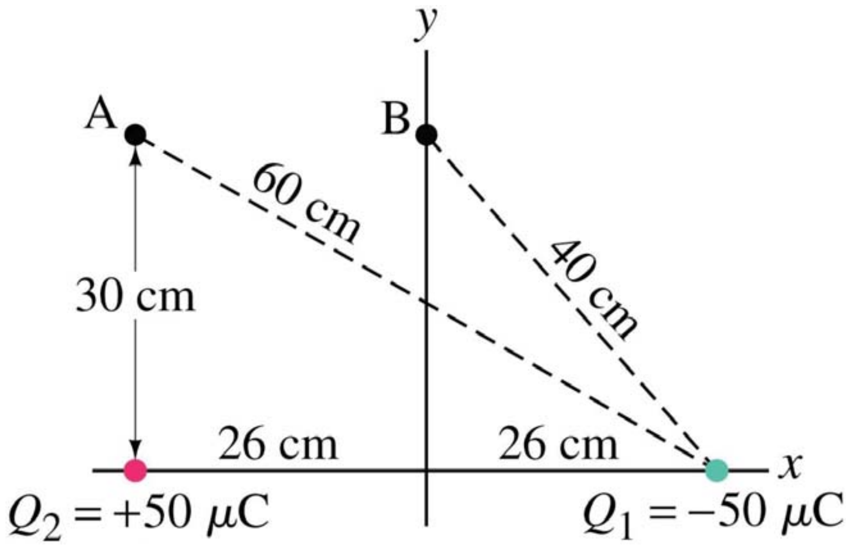
Example 06

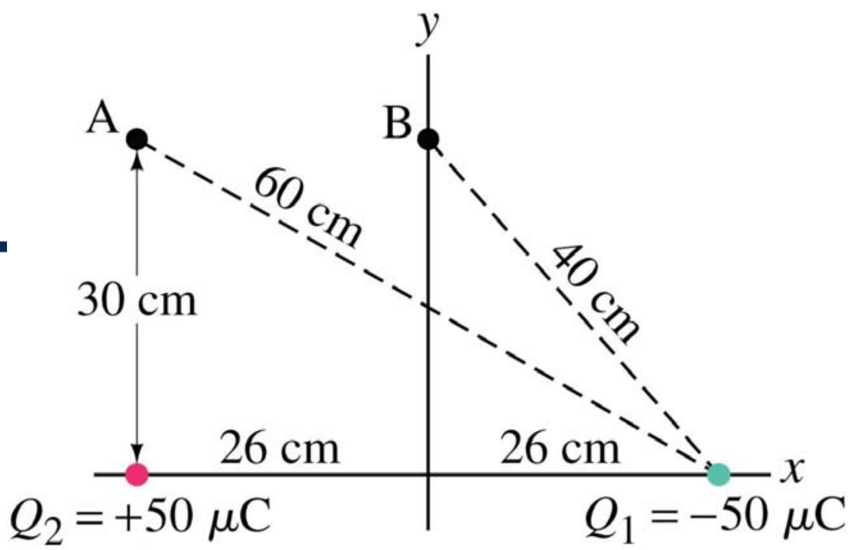
- **Work required to bring two positive charges together.**
- What minimum work must be done by an external force to bring a charge q = 3.00 *u.C* from a great distance away (take $r = \infty$) to a point 0.500 m from a charge Q = 20.0 *u.C?*

- $\Delta w = -Q(v_b - va) = -Q\left(\frac{k\,q}{r_b} - \frac{k\,q}{r_a}\right)$
- $r_a = d \;\&\; r_b = \infty$
- $\Delta w = k\,\frac{Qq}{d}$
- $\Delta w = 8.99 \times 10^9 \times \frac{3 \times 10^{-6} \times 20 \times 10^{-6}}{0.5} = 1.08\,j$

Example 07

- Potential above two charges.
- Calculate the electric potential (a) at point A in the figure due to the two charges shown.



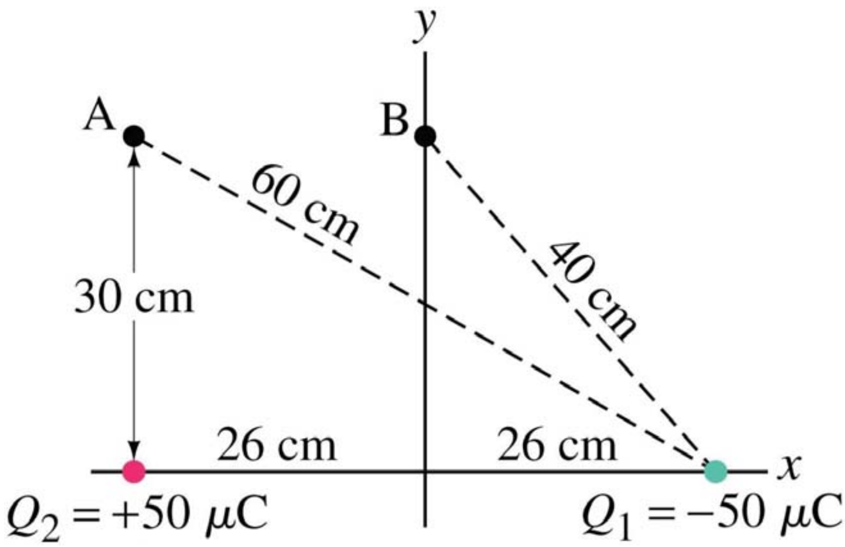


- $V_a = VA_2 + VA_1 = k\frac{Q_2}{r_{2A}} + k\frac{Q_1}{r_{1A}}$
- $= 8.99 \times 10^9 \left(\frac{50}{0.3} + \frac{-50}{0.26}\right) = 7.5 \times 10^5\ V$

MCQ

- What is the electric potential at point B?

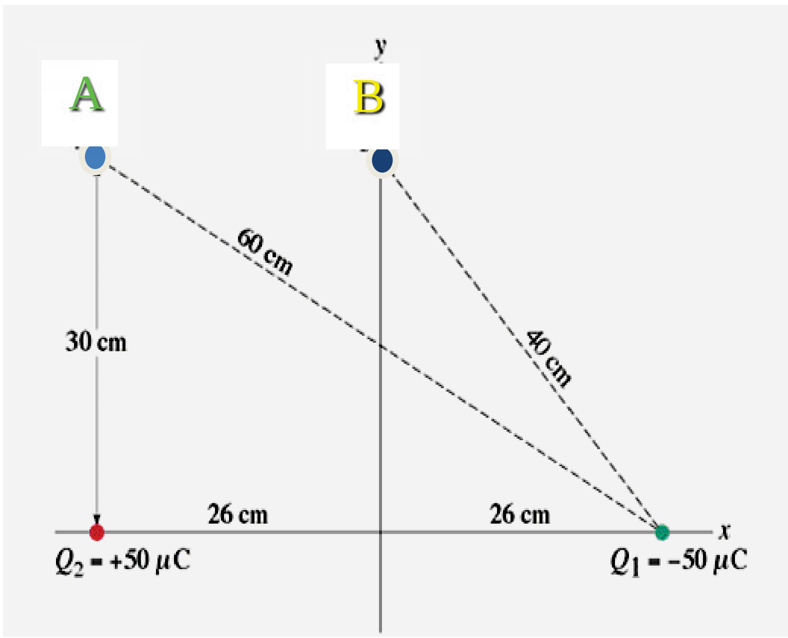
- 1) $V > 0$
- 2) $V = 0$
- 3) $V < 0$



MCQ

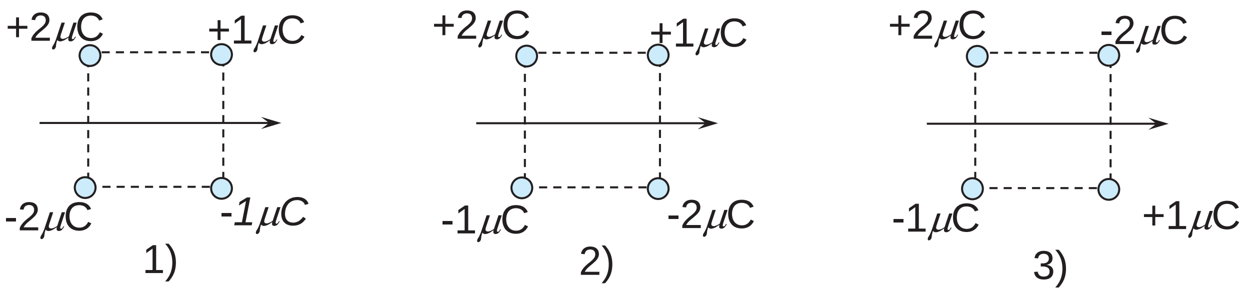
- What is the electric potential at point B?

- 1) $V > 0$
- 2) $V = 0$
- 3) $V < 0$



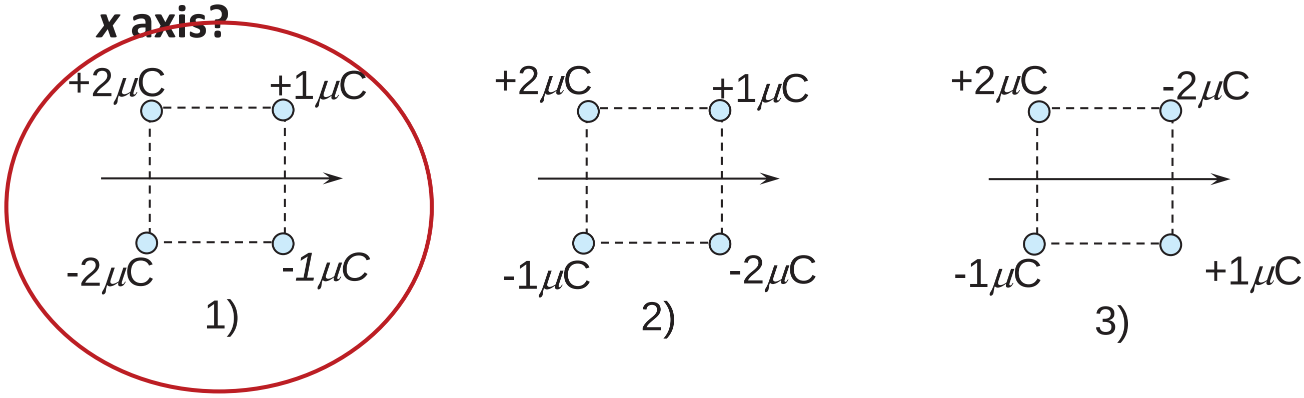
MCQ

- **Which of these configurations gives *V* = 0 at all points on the *x* axis?**



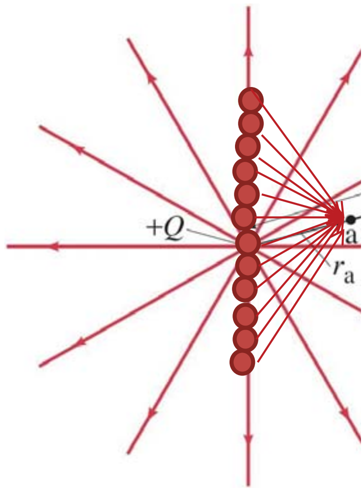
MCQ

- **Which of these configurations gives *V* = 0 at all points on the *x* axis?**



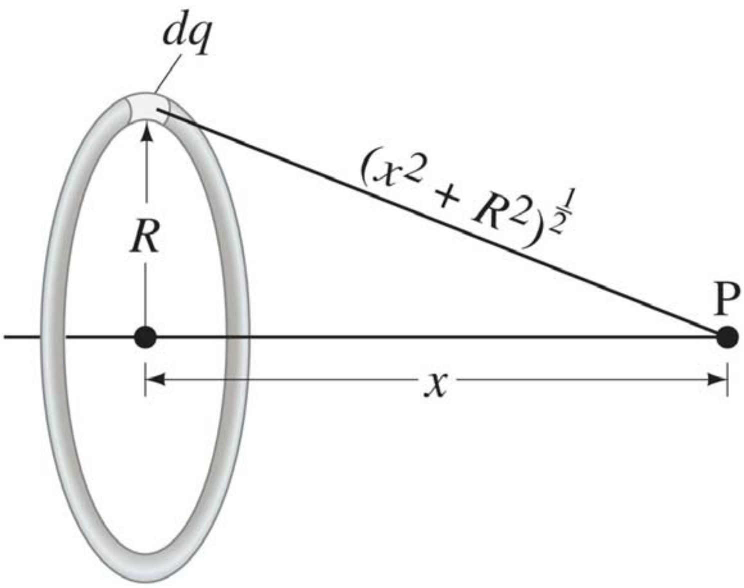
Potential Due to Any Charge Distribution

- The potential due to an arbitrary charge distribution can be expressed as a sum or integral (if the distribution is continuous):
- $v = \frac{Q}{4\pi\varepsilon_0 r}$ $(\mathrm{v} = 0 \; @ \; \mathrm{r} = \infty)$
- $v = \sum_{i=1}^{n} v_i = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_{ia}}$
- $v = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r}$

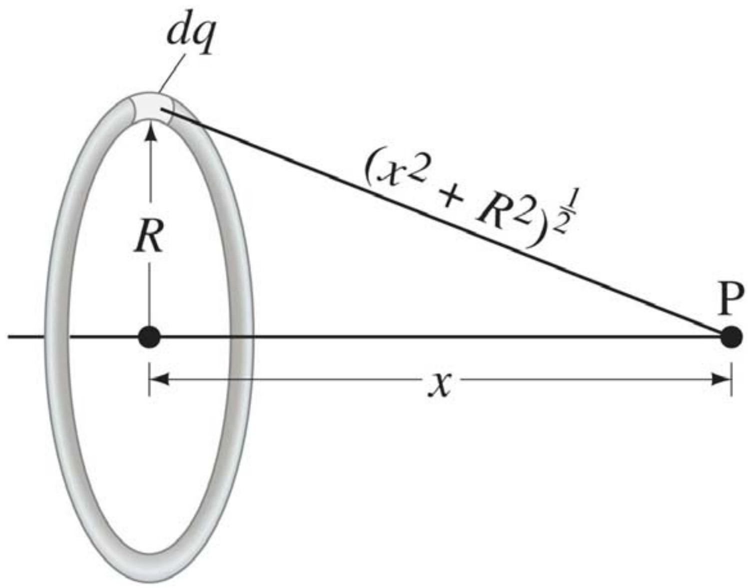


Example 08

- Potential due to a ring of charge.
- A thin circular ring of radius R has a uniformly distributed charge Q. Determine the electric potential at a point P on the axis of the ring a distance x from its center.

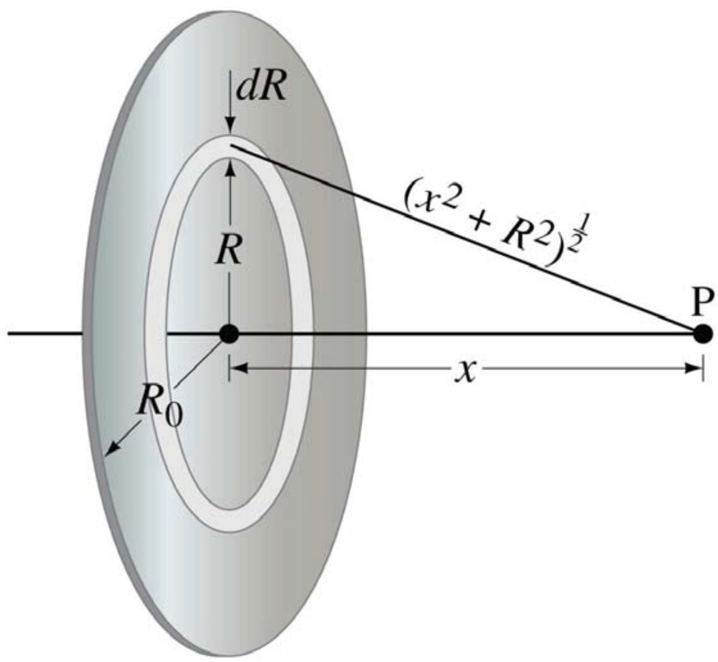


- $v = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r}$
- $= \frac{1}{4\pi\varepsilon_0} \frac{1}{\sqrt{x^2 + R^2}} \int dq$
- $= \frac{1}{4\pi\varepsilon_0} \frac{Q}{\sqrt{x^2 + R^2}}$

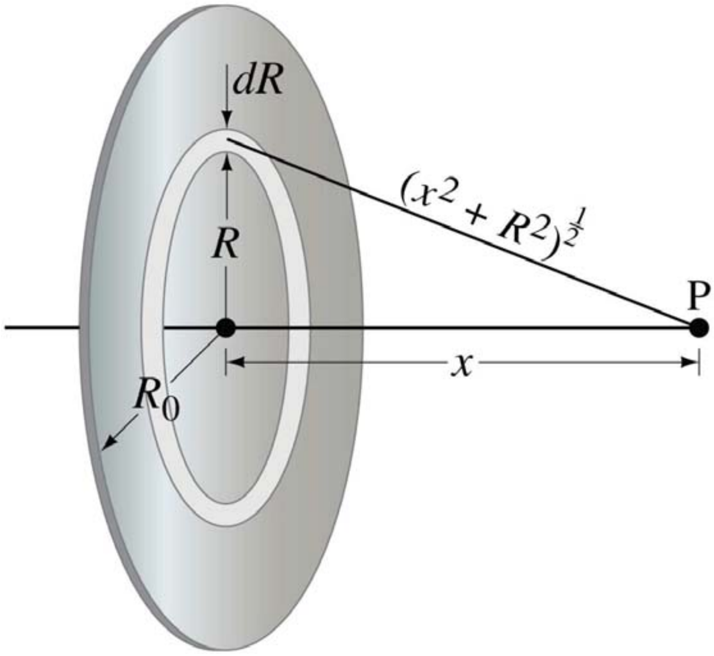


Example 09

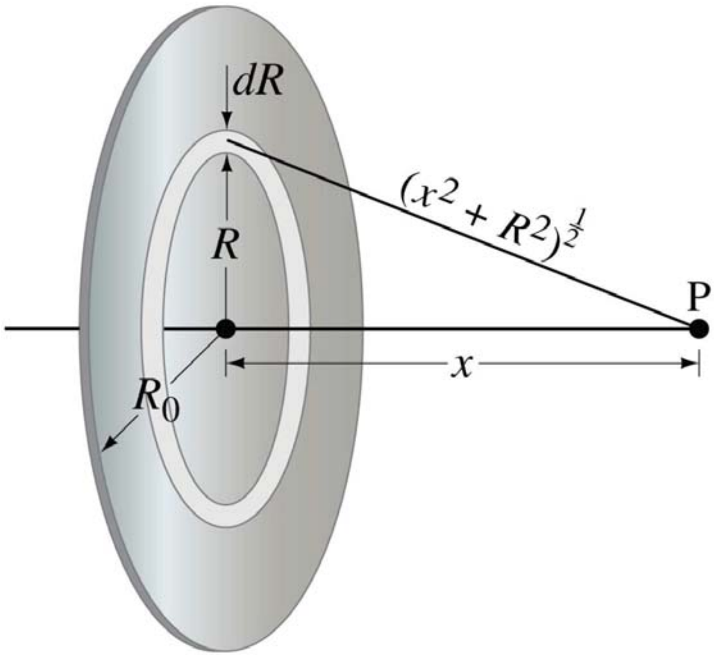
- Potential due to a charged disk.
- A thin flat disk, of radius R_0, has a uniformly distributed charge Q. Determine the potential at a point P on the axis of the disk, a distance x from its center



- $A = \pi {R_0}^2$
- $dA = 2\pi\, R\, dR$
- $\sigma = {}^{Q}/_{\pi\, {R_0}^2}$
- $\sigma = {}^{dQ}/_{2\pi\, R\, dR}$
- $\frac{dQ}{Q} = \frac{2\, \pi\, R\, dR}{\pi\, {R_0}^2}$
- $dQ = \frac{2\, Q\, R\, dR}{{R_0}^2}$
- $V = \frac{1}{4\, \pi\, \varepsilon_0} \int \frac{dQ}{\sqrt{x^2+R^2}}$
- $V = \frac{2Q}{4\, \pi\, \varepsilon_0 R_0} \int_0^{R_0} \frac{R\, dR}{\sqrt{x^2+R^2}}$

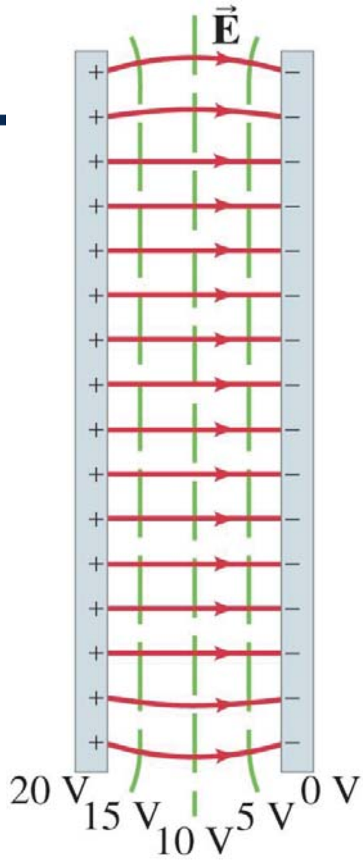


- $V = \frac{Q}{2\pi\, \varepsilon_0 R_0} \int_0^{R_0} \frac{R\, dR}{\sqrt{x^2+R^2}}$
- $V = \frac{Q}{2\pi\, \varepsilon_0 R_0} \left(\sqrt{x^2 + R^2}\right) \begin{matrix} R = R_0 \\ R = 0 \end{matrix}$
- $V = \frac{Q}{2\pi\, \varepsilon_0 R_0} \left[\sqrt{x^2 + {R_0}^2} - x\right]$

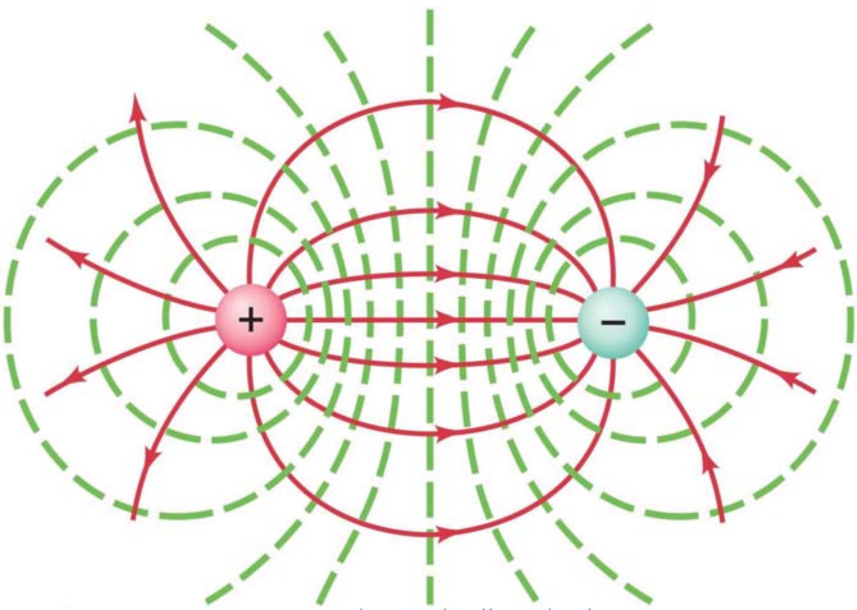


Equipotential Surfaces

- An equipotential is a line or surface over which the potential is constant.
- Electric field lines are perpendicular to equipotential.
- The surface of a conductor is an equipotential
- $(E_{//} = 0 \rightarrow \partial V / \partial s_{//} = 0)$
- Equipotential lines (the green dashed lines) between two oppositely charged parallel plates. Note that they are perpendicular to the electric field lines (solid red lines).



- Equipotential lines (green, dashed) are always perpendicular to the electric field lines (solid red) shown here for two equal but oppositely charged particles.




Thanks,..
See you next week (ISA),...
٥٣
Dr. Ahmed ElShafee, ACU : Spring 2015, Physics II